APPLIED MATHEMATICS

The Department of Applied Mathematics in the College of Arts and Sciences offers a range of courses and research opportunities in many areas, including computational mathematics, mathematical biology, nonlinear phenomena, physical applied mathematics, and probability and statistics. Each of these areas is described below.

Course code for this program is APPM.

Computational Mathematics

The study of computational mathematics has grown rapidly in recent years and has allowed scientists and engineers to answer questions and to develop insights not possible just a decade or two ago. Modern computational methods require in-depth knowledge of a variety of mathematical subjects including linear algebra, analysis, ordinary and partial differential equations, asymptotic analysis, elements of harmonic analysis and nonlinear equations. Since computers are invaluable tools for an applied mathematician, students are expected to attain a high level of computer literacy and to gain a substantial knowledge of operating systems and hardware. Computational mathematics courses include the study of computational linear algebra, optimization, numerical solution of ordinary and partial differential equations, solution of nonlinear equations and advanced seminars in wavelet and multiresolution analysis and in multigrid methods, radial basis functions and algorithm design and development, more generally.

Mathematical Biosciences

Advances in our ability to quantitatively study biological phenomena have provided a number of exciting opportunities for applied mathematicians. The careful modeling, analysis and simulation of these systems using the standard and state-of-the-art tools of applied mathematics has led to novel and non-intuitive insights into biology. Furthermore, deeper understanding of the inherently complex and multiscale nature of biological systems, in many cases, requires the development of new mathematical tools, techniques and methodologies (a challenge to which applied mathematics is particularly well suited). For students interested in pursuing research in mathematical biology, good preparatory classes would include differential equations, advanced calculus, numerical analysis and probability and statistics, as well as supplemental courses in the appropriate biological, biomedical or bioengineering fields. Research areas at CU encompass immunology, virology, bacteriology, population genetics and cardiac nonlinear dynamics. Specifically, current topics of interest include model selection and control of in vivo HIV pathogenesis dynamics, modeling of intracellular calcium dynamics, the analysis of heart rhythm instabilities, the role of aggregation and fragmentation in bacteremia and bacterial pneumonia, inverse problems arising in the use of population genetics and bioinformatics to identify geographic features and the analysis of patterns in biological sequences such as DNA and RNA.

Dynamical Systems and Nonlinear Phenomena

In recent years, there has been an explosion of interest in the study of nonlinear waves and dynamical systems with analytical results, often motivated by the use of computers. The faculty in the Department of Applied Mathematics are actively and intensively involved in this growing field. Research areas include qualitative analysis and computational dynamics, conservative and dissipative systems, bifurcation theory, the onset and development of chaos, wavelets and multiresolution analysis, integrable systems, solitons, cellular automata, analytic dynamics, pattern formation and symmetry, synchronization, dynamics on networks, fluid dynamics, transport and mixing and the study of nonlinear phenomena arising from the interactions of many interconnected dynamical units. Department courses in this field include dynamical systems, nonlinear wave motion and many advanced seminars. Suitable background courses are analysis, computation and methods in applied mathematics. Valuable supplemental courses include mechanics and fluid dynamics.

Physical Applied Mathematics

Physical applied mathematics is a term that generally refers to the study of mathematical problems with direct physical application. This area of research is intrinsically interdisciplinary. In addition to mathematical analysis, it requires an in-depth understanding of the underlying applications area, and usually requires knowledge and experience in numerical computation. The department has approximately 40 affiliated faculty who can direct thesis research in areas such as atmospheric and fluid dynamics, theoretical physics, plasma physics, genetic structure, parallel computation, etc. The department’s course requirements are designed to provide students with a foundation for their study (analysis and computation). The department also requires supplemental courses in one of the sciences or engineering fields necessary for thesis research in physical applied mathematics.

Statistics and Applied Probability

Almost all natural phenomena in the technological, biological, physical and social sciences have random components with complex levels of interactions, part stochastic, part deterministic. Applied probability is the application of probabilistic and analytic methods to model, understand and predict the behavior of real-life problems that involve random elements. Statistics is the science of using data that typically arise from the randomness inherent in nature to gain new knowledge. Areas of current interest by applied math and their affiliated faculty include optimization of stochastic networks; the study of stochastic processes, and stochastic differential equations in hydrology and telecommunications; probabilistic models, nonparametric regression methods, shrinkage estimation, gene expression microarray data analysis, false discovery rate control, classification methods and statistical tests based on these models, in genetics and RNA sequencing; and extreme value theory in estimation of maximal wind speeds. Appropriate course work includes analysis, stochastic processes, simulation techniques and mathematical statistics, as well as background courses in one of the sciences or engineering fields in which one intends to do research.

For details on the range of courses and research opportunities available through the Department of Applied Mathematics, visit www.colorado.edu/amath.

Master's Degree

• Applied Mathematics - Master of Science (MS) (catalog.colorado.edu/graduate/colleges-schools/arts-sciences/programs-study/applied-mathematics/applied-mathematics-master-science-ms)

Doctoral Degree

• Applied Mathematics - Doctor of Philosophy (PhD) (catalog.colorado.edu/graduate/colleges-schools/arts-sciences/
Faculty

While many faculty teach both undergraduate and graduate students, some instruct students at the undergraduate level only. For more information, contact the faculty member’s home department.

Ablowitz, Mark J (https://experts.colorado.edu/display/fisid_100691)
Professor; PhD, Massachusetts Institute of Technology

Appelö, Daniel
Associate Professor; PhD, Royal Institute of Technology, Stockholm, Sweden

Bebernes, Jerrold
Professor Emeritus

Becker, Stephen R (https://experts.colorado.edu/display/fisid_154263)
Assistant Professor; PhD, California Institute of Technology

Beylkin, Gregory (https://experts.colorado.edu/display/fisid_100437)
Professor; PhD, New York University

Bhat, Yermal Sujeet (https://experts.colorado.edu/display/fisid_146506)
Instructor; PhD, University of Florida

Bortz, David Matthew (https://experts.colorado.edu/display/fisid_143348)
Associate Professor; PhD, North Carolina State University at Raleigh

Corcoran, Jem (https://experts.colorado.edu/display/fisid_118142)
Associate Professor; PhD, Colorado State University

Cox, Murray William (https://experts.colorado.edu/display/fisid_153192)
Instructor; PhD, Texas AM University

Curry, James H (https://experts.colorado.edu/display/fisid_105730)
Professor; PhD, University of California-Berkeley

Dougherty, Anne Margaret (https://experts.colorado.edu/display/fisid_101349)
Senior Instructor; PhD, University of Wisconsin-Madison

Dukic, Vanja (https://experts.colorado.edu/display/fisid_148718)
Professor; PhD, Brown University

Easton, Robert
Professor Emeritus

Fornberg, Bengt (https://experts.colorado.edu/display/fisid_108048)
Professor; PhD, Univ of Uppsala (Sweden)

Grooms, Ian G (https://experts.colorado.edu/display/fisid_155588)
Assistant Professor; PhD, University of Colorado Boulder

Hoefer, Mark (https://experts.colorado.edu/display/fisid_154264)
Associate Professor; PhD, University of Colorado Boulder

Huang, Yu-Jui (https://experts.colorado.edu/display/fisid_157746)
Assistant Professor; BBA, National Taiwan University

Julien, Keith (https://experts.colorado.edu/display/fisid_108913)
Professor; PhD, University of Cambridge (England)

Kilpatrick, Zachary Peter (https://experts.colorado.edu/display/fisid_155782)
Assistant Professor; PhD, University of Utah

Kleiber, William Paul (https://experts.colorado.edu/display/fisid_151943)
Assistant Professor; PhD, University of Washington

Li, Congming (https://experts.colorado.edu/display/fisid_100647)
Professor; PhD, New York University

Lladser, Manuel Bosco (https://experts.colorado.edu/display/fisid_134170)
Associate Professor; PhD, Ohio State University

Lyles, Danielle
Instructor; PhD, Cornell University, Ithaca, NY, USA

Manteuffel, Thomas A (https://experts.colorado.edu/display/fisid_102137)
Professor Emeritus; PhD, University of Illinois at Urbana-Champaign

Mc Cormick, Steven
Professor Emeritus

Meiss, James D (https://experts.colorado.edu/display/fisid_103702)
Professor; PhD, University of California-Berkeley

Norris, Jan Adam (https://experts.colorado.edu/display/fisid_101412)
Senior Instructor; PhD, University of Colorado Boulder

Restrepo, Juan G (https://experts.colorado.edu/display/fisid_145811)
Associate Professor; PhD, University of Maryland College Park Campus

Segur, Harvey (https://experts.colorado.edu/display/fisid_102287)
Professor; PhD, University of California-Berkeley

Zaharatos, Brian R (https://experts.colorado.edu/display/fisid_156225)
Instructor; MS, Colorado School of Mines

Courses

**APPM 5120 (3) Introduction to Operations Research**
Studies linear and nonlinear programming, the simplex method, duality, sensitivity, transportation and network flow problems, some constrained and unconstrained optimization theory, and the Kuhn-Tucker conditions, as time permits. Department enforced prerequisite: APPM 2130 or MATH 2135 or MATH 2135.

**Equivalent - Duplicate Degree Credit Not Granted:** APPM 4120 and MATH 4120 and MATH 5120

**Requisites:** Restricted to graduate students only.

**APPM 5350 (3) Methods in Applied Mathematics: Fourier Series and Boundary Value Problems**
Department enforced prerequisite courses: APPM 2350 or MATH 2400 and APPM 2360 and a prerequisite or corequisite course: APPM 3310 or MATH 2130 or MATH 2135.

**Equivalent - Duplicate Degree Credit Not Granted:** APPM 4350

**Requisites:** Restricted to graduate students only.
APPM 5360 (3) Methods in Applied Mathematics: Complex Variables and Applications
Introduces methods of complex variables, contour integration and theory of residues. Applications include solving partial differential equations by transform methods, Fourier and Laplace transforms and Reimann-Hilbert boundary-value problems, conformal mapping to ideal fluid flow and/or electrostatics. Department enforced prerequisites: APPM 2350 or MATH 2400 and APPM 2360 and a prerequisite or corequisite course of APPM 3310 or MATH 3130 or MATH 3135.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4360
Requisites: Restricted to graduate students only.

APPM 5380 (3) Modeling in Applied Mathematics
An exposition of a variety of mathematical models arising in the physical and biological sciences. Students' modeling projects are presented in class. Topics may include: GPS navigation, medical imaging, ocean waves, and computerized facial recognition. Department enforced prerequisites: APPM 2350 or MATH 2400 and APPM 2360.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4380
Requisites: Restricted to graduate students only.
Recommended: Prerequisites APPM 3310 and APPM 4350 and APPM 4650.

APPM 5390 (3) Modeling in Mathematical Biology
Investigates how complex systems in biology can be studied using applied mathematics. Examines several case studies which include topics from microbiology, enzyme reaction kinetics, neuroscience, ecology, epidemiology, physiology and bioengineering. Department enforced prerequisites: APPM 2360 and APPM 3310 or MATH 2130 or MATH 2135 or instructor consent required.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4390
Requisites: Restricted to graduate students only.

APPM 5430 (3) Methods in Applied Mathematics: Applications of Complex Variables
Reviews basic ideas of complex analysis, including solutions of ODEs and PDEs of physical interest via complex analysis; conformal mapping, including Schwarz-Christoffel transformations and generalizations; computational methods; Riemann-Hilbert problems; topics in asymptotic methods. Department enforced prerequisite: APPM 4360 or APPM 5360.
Requisites: Restricted to graduate students only.

APPM 5440 (3) Applied Analysis 1
Discusses the elements of basic real and complex analysis, Banach spaces, Lp spaces and many relevant inequalities. Includes applications of existence and uniqueness to solutions of various types of ordinary differential equations, partial differential equations, and integral equations. Department enforced prerequisites: APPM 4440 and APPM 4450.
Requisites: Restricted to graduate students only.

APPM 5450 (3) Applied Analysis 2
Continuation of APPM 5440. Department enforced prerequisite: APPM 5440.
Requisites: Restricted to graduate students only.

APPM 5460 (3) Methods in Applied Mathematics: Dynamical Systems and Differential Equations
Introduces the theory and applications of dynamical systems through solutions to differential equations. Covers existence and uniqueness theory, local stability properties, qualitative analysis, global phase portraits, perturbation theory and bifurcation theory. Special topics may include Melnikov methods, averaging methods, bifurcations to chaos and Hamiltonian systems. Department enforced prerequisites: APPM 2360 and APPM 3310 and APPM 4440.
Requisites: Restricted to graduate students only.

APPM 5470 (3) Methods of Applied Mathematics: Partial Differential and Integral Equations
Requisites: Restricted to graduate students only.

APPM 5480 (3) Methods of Applied Mathematics: Approximation Methods
Covers asymptotic evaluation of integrals (stationary phase and steepest descent), perturbation methods (regular and singular methods, and inner and outer expansions), multiple scale methods and applications to differential and integral equations. Department enforced prerequisite: APPM 5470.
Requisites: Restricted to graduate students only.

APPM 5500 (3) Statistical Collaboration
Educates and trains students to become effective interdisciplinary collaborators by developing the communication and collaboration skills necessary to apply technical statistics and data science skills to help domain experts answer research questions. Topics include structuring effective meetings and projects; communicating statistics to non-statisticians; using peer feedback, self-reflection and video analysis to improve collaboration skills; creating reproducible statistical workflows; working ethically.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4500
Requisites: Requires a prerequisite course of APPM 5520 (minimum grade C).
Grading Basis: Letter Grade

APPM 5505 (2) Advanced Statistical Collaboration
Educates and trains students to become advanced interdisciplinary collaborators by developing and refining the communication, collaboration and technical statistics and data science skills necessary to collaborate with domain experts to answer research questions. Students work on multiple projects. Discussions center on technical skills necessary to solve research problems and video analysis to improve communication and collaboration skills.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4505
Requisites: Requires prerequisite course of APPM 4500 or APPM 5500 (minimum grade C).
Grading Basis: Letter Grade
Requisites:
Equivalent - Duplicate Degree Credit Not Granted:
Gain insight into the processes under study.
Monte Carlo simulation of random variables throughout the semester to study systems are studied with some introduction to networks of queues. Uses Poisson point processes. Queuing theory, terminology and single queue analysis of Markov chains, both discrete and continuous time, including Brief review of conditional probability and expectation followed by a
APPM 5560 (3) Markov Processes, Queues, and Monte Carlo Simulations
Grading Basis: Letter Grade
APPM 5550 (3) Spatial Statistics
Introduces the theory of spatial statistics with applications. Topics include basic theory for continuous stochastic processes, spatial prediction and kriging, simulation, geostatistical methods, likelihood and Bayesian approaches, spectral methods and an overview of modern topics such as nonstationary models, hierarchical modeling, multivariate processes, methods for large datasets and connections to spines.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4550
Requisites: Requires prerequisite course of APPM 4520 or APPM 5520 or MATH 4520 or MATH 5520 (minimum grade C).
Grading Basis: Letter Grade
APPM 5560 (3) Markov Processes, Queues, and Monte Carlo Simulations
Brief review of conditional probability and expectation followed by a study of Markov chains, both discrete and continuous time, including Poisson point processes. Queuing theory, terminology and single queue systems are studied with some introduction to networks of queues. Uses Monte Carlo simulation of random variables throughout the semester to gain insight into the processes under study.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4560
Requisites: Restricted to graduate students only.
APPM 5570 (3) Statistical Methods
Covers basic statistical concepts with accompanying introduction to the R programming language. Topics include discrete and continuous probability laws, random variables, expectation and variance, central limit theorem, testing hypothesis and confidence intervals, linear regression analysis, simulations for validation of statistical methods and applications of methods in R.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4570
Requisites: Restricted to graduate students only.
APPM 5580 (3) Introduction to Statistical Learning
Consists of applications and methods of statistical learning. Consists of multiple linear regression, classification, regularization, splines, tree-based methods, support vector machines and unsupervised learning.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4580
Requisites: Requires prerequisite course of APPM 4570 or APPM 5570 (minimum grade C). Restricted to graduate students only.
APPM 5590 (3) Statistical Modeling
Introduces methods, theory and applications of statistical models, from linear models (simple and multiple linear regression), to hierarchical linear models. Topics such as estimation, residual diagnostics, goodness of fit, transformations, and various strategies for variable selection and model comparison will be discussed in depth. Examples and exercises will be demonstrated using statistical software. Department enforced prerequisite: APPM 4570 or APPM 4520 or MATH 4520 or instructor consent required.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4590
Requisites: Restricted to graduate students only.
APPM 5600 (3) Numerical Analysis 1
Solution of nonlinear algebraic equations, interpolation, integration, approximation, and numerical linear algebra. Department enforced prerequisite: APPM 3310 or MATH 2130 and experience with a scientific programming language.
Requisites: Restricted to graduate students only.
APPM 5610 (3) Numerical Analysis 2
Numerical linear algebra, eigenvalue problems, optimization problems, and ordinary and partial differential equations. Department enforced prerequisite: APPM 5600 or MATH 5600.
Requisites: Restricted to graduate students only.
APPM 5720 (1-3) Open Topics in Applied Mathematics
Provides a vehicle for the development and presentation of new topics that may be incorporated into the core courses in applied mathematics. Department enforced prerequisite: variable, depending on the topic, see instructor.
Equivalent - Duplicate Degree Credit Not Granted: APPM 4720
Repeatable: Repeatable for up to 6.00 total credit hours. Allows multiple enrollment in term.
Requisites: Restricted to graduate students only.
APPM 6470 (3) Advanced Partial Differential Equations
Continuation of APPM 5470. Advanced study of the properties and solutions of elliptic, parabolic, and hyperbolic partial differential equations. Topics include the study of Sobolev spaces and variational methods as they relate to PDEs, and other topics as time permits. Department enforced prerequisite: APPM 5470.
Requisites: Restricted to graduate students only.
APP 6520 (3) Mathematical Statistics
Emphasizes mathematical theory of statistics. Topics include distribution theory, estimation and testing of hypotheses, multivariate analysis, and nonparametric inference, all with emphasis on theory. Department enforced prerequisite: APPM 5520 or MATH 5520.
Requisites: Restricted to graduate students only.

APP 6550 (3) Introduction to Stochastic Processes
Systematic study of Markov chains and some of the simpler Markov processes including renewal theory, limit theorems for Markov chains, branching processes, queuing theory, birth and death processes, and Brownian motion. Applications to physical and biological sciences.
Department enforced prerequisite: MATH 4001 or MATH 4510 or APPM 3570 or APPM 4560 or instructor consent.
Equivalent - Duplicate Degree Credit Not Granted: MATH 6550
Requisites: Restricted to graduate students only.

APP 6610 (3) Introduction to Numerical Partial Differential Equations
Covers finite difference, finite element, finite volume, pseudo-spectral, and spectral methods for elliptic, parabolic, and hyperbolic partial differential equations.
Department enforced prerequisite: APPM 5600.
Requisites: Restricted to graduate students only.
Recommended: Prerequisite APPM 5610 or graduate numerical linear algebra.

APP 6640 (3) Multigrid Methods
Develops a fundamental understanding of the principles and techniques of the multigrid methodology, which is a widely used numerical approach for solving many problems in such diverse areas as aerodynamics, astrophysics, chemistry, electromagnetics, hydrology, medical imaging, meteorology/oceanography, quantum mechanics, and statistical physics.
Requisites: Restricted to graduate students only.

APP 6900 (1-6) Independent Study
Introduces graduate students to research foci of the Department of Applied Mathematics.
Requisites: Restricted to graduate students only.

APP 6940 (1) Master's Degree Candidate
Requisites: Restricted to graduate students only.
Grading Basis: Pass/Fail

APP 6950 (1-6) Master's Thesis
Repeatable: Repeatable for up to 6.00 total credit hours.
Requisites: Restricted to graduate students only.

APP 6900 (1-3) Topics in Applied Mathematics
Provides a vehicle for the development and presentation of new topics with the potential of being incorporated into the core courses in applied mathematics.
Repeatable: Repeatable for up to 6.00 total credit hours. Allows multiple enrollment in term.
Requisites: Restricted to graduate students only.

APP 7900 (1-3) Independent Study
Introduces graduate students to research foci of the Department of Applied Mathematics.
Requisites: Restricted to graduate students only.

APP 8000 (1) Colloquium in Applied Mathematics
Introduces graduate students to the major research foci of the Department of Applied Mathematics.
Requisites: Restricted to Applied Mathematics (APPM) graduate students only.

APP 8100 (1) Seminar in Dynamical Systems
Introduces advanced topics and research in dynamical systems.
Requisites: Restricted to Applied Mathematics (APPM) graduate students only.

APP 8300 (1-3) Nonlinear Waves Seminar
Introduces the core methods in the analysis of nonlinear partial differential and integral equations or systems to graduate students. Provides a vehicle for the development, presentation, and corporate research of new topics in PDE and analysis.
Requisites: Requires prerequisite course of APPM 5440 (minimum grade D-). Restricted to Applied Mathematics (APPM) graduate students only.

APP 8400 (1) Mathematical Biology Seminar
Introduces advanced topics and research in mathematical and computational biology.
Instructor consent required.
Requisites: Restricted to graduate students only.
Grading Basis: Letter Grade

APP 8500 (1) Statistics, Optimization and Machine Learning Seminar
Research-level seminar that explores the mathematical foundations of machine learning, in particular how statistics and optimization give rise to well-founded and efficient algorithms.
Requisites: Restricted to graduate students only.
Grading Basis: Letter Grade

APP 8600 (1) Seminar in Computational Mathematics
Introduces advanced topics and research in computational mathematics.
Requisites: Restricted to Applied Mathematics (APPM) graduate students only.

APP 8990 (1-10) Doctoral Dissertation
All doctoral students must register for no fewer than 30 hours of dissertation credit as part of the requirements for the degree. No more than 10 credit hours may be taken in any one semester.
Repeatable: Repeatable for up to 30.00 total credit hours.
Requisites: Restricted to graduate students only.